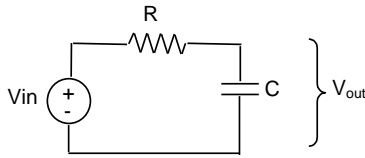


Mechanical Systems Laboratory: Lecture 9

Analysis of a 1st-order, Low-Pass Filter Circuit in the Time and Frequency Domains

The following circuit is a low-pass filter. It is useful to clean up signals with high frequency noise on them:



1. Time Domain Analysis

Let's analyze the response of this circuit to a step input

We'll use the method of undetermined coefficients to solve the differential equation. You can remember this very useful technique for linear, ordinary, differential equations using the following mnemonic:

1. Generals: set the forcing function = 0 and find the general solution to homogenous equation (don't evaluate it's coefficient yet)
2. are Particular: find the particular solution (assume particular soln is same form as forcing function)
3. about Initial Conditions: sum the homogenous and particular solutions and solve for the coefficient to the homogenous equation that satisfies the initial conditions.

Summary of important concepts:

- Method of undetermined coefficients for solving a differential equation.
- Time constant: a 1st order system has gone 63% of the way to its final value after one time constant – standard engineering technique for quantifying “how fast” a system responds.

2. Frequency Domain Analysis

Let's analyze how this system responds to a sinusoidal input. Remember: sine in \Rightarrow sine out (scaled and shifted), for a linear system. We will use three methods to find the scaling and shifting.

Method 1. Solve differential equation using method of undetermined coefficients (difficult)

Homogenous solution:

Particular solution:

Useful trig. identity: $A \cos(\theta) + B \sin(\theta) = \sqrt{A^2 + B^2} \sin(\theta + \tan^{-1}(\frac{A}{B}))$

Method 2: Take Laplace Transform of differential equation that describes circuit, find the transfer function, and solve for frequency response (easier than Method 1)

Brief review of complex variables:

Complex variables keep track of two pieces of information, real and imaginary part, or magnitude and phase

Can think of complex variables as a point in the complex plane.

Can write point in Cartesian or polar coordinates.

To find the magnitude in Cartesian form:

To find the phase in Cartesian form:

Magnitude of two complex variables divided by each other:

Phase of two complex variables divided by each other:

Now, find the transfer function and frequency response:

Method 3: Use “impedances” to find transfer function (easiest)

Circuit element	Time domain	Frequency domain	Impedance
Resistor	$V(t) = R I(t)$	$V(s) = R I(s)$	R
Capacitor	$V(t) = \frac{1}{C} \int i(t)$		
Inductor	$V(t) = L di/dt$		

Note: All the usual circuit rules still hold in the frequency domain because of superposition (KVL, KCL, Op amp rules, voltage divider...). So, treat impedances like (frequency dependent) resistors in finding a circuit's transfer function.

What do the magnitude response (i.e. scaling or attenuation factor) and phase shift response actually look like?

Fill in the following chart:

	Magnitude or Scaling	Phase
Small ω		
$\omega = 1/RC = 1/\tau$		
$\omega \Rightarrow \text{infinity}$		

The frequency $1/\tau$ is called the “corner frequency” or “bandwidth” of the system. For this low-pass filter, input sinusoids with a frequency higher than the bandwidth are “filtered” or “attenuated”.

Summary of important concepts:

- How to find a transfer function and the frequency response
- Impedances
- Corner frequency